Fig. 1.

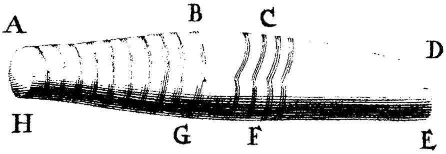


Fig. 2.

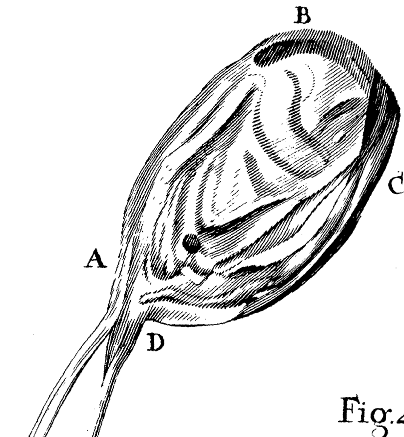


Fig. 3.

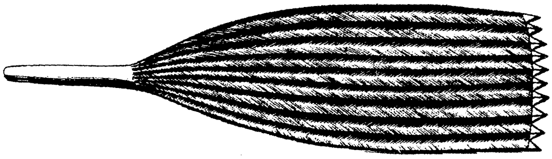


Fig. 4.

Fig. 5.

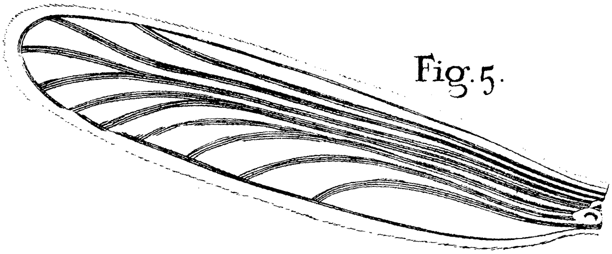


Fig. 6.

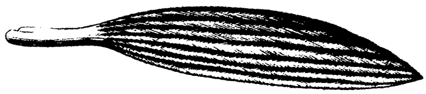
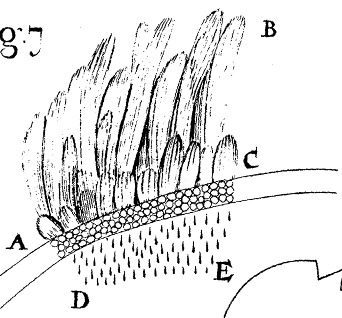


Fig. 7



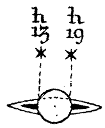


Fig. 8.

Fig. 9.

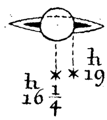


Fig. 10

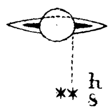


Fig. 11.

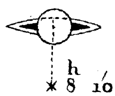
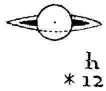


Fig. 12.



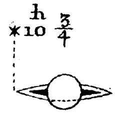
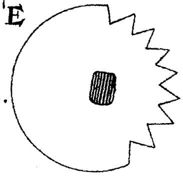


Fig. 13.

Fig. 14.



A Correction of the Theory *of the Motion of the* Satelite *of* Saturn, *by that Ingenious* Aſtronomer Mr. EDMUND HALLY.

SIR,

I here ſend you an *Aſtronomical account* of the moſt remote of all the *Planets* of our *Vortex*, and withal of the moſt inconſiderable, I mean of the *Satellite* of *Saturn*, diſcovered in the year 1655 by *Monſieur Chriſtian Hugens* of *Zulichem*, who in that accurate Treatiſe of his *Syſtema Saturnium*, from *pag.* 25 to 34 gives us the *Theory* of its *Motion* as well as the ſhortneſs of the interval of time between his *Obſervations* would admit; and ſince him I know none that have gone about to improve the ſaid *Theory*.

The late *Conjunctions* of *Jupiter* and *Saturn* giving me frequent occaſions of viewing them both, with a *Teleſcope* that I have of about 24 foot, and pretty good of that length, I eaſily remarqued this *Satallite* of *Saturn*, and haveing found it, in a convenient poſition to determine its place, I percieved that *Hugens*'s numbers were conſiderably run out, and about 15 *degrees* in twenty years too ſwift; this made me reſolve more nicely to enquire into its *period*; and accordingly I waited till I had gotten a competent number of *Obſervations*, the moſt conſiderable whereof are theſe.

1682. *November* 13° 13ʰ 00′ PM. the *Satellite* appeared on the *North* ſide of *Saturn*, and a *perpendicular* let fall from it on the tranſverſs *diameter* of the *Ring*, fell upon the middle of the dark ſpace of the following *Anſa*; and the ſame night 19ʰ 00′ it had paſt the *Conjunction*, and the *perpendicular* fell exactly on the *Weſtern* edg of the *Globe* of *Saturn*, as in *Tab.* 2. *Figure* 8. The *Northern* Latitude, and retrograde motion, made it evident that the *Satellite* was then in *Perigæo*.

Again

Again *November* 21°. 16^h. 15'. this *Satellite* of *Saturn* was on his *South* side, the *perpendicular* on the line of the *Ansæ* fell on the middle of the dark space of the *Western Ansæ*, and the same night 19^h 00' the *perpendicular* fell precisly on the *Center* of *Saturn*, and the distance therefrom was somewhat less than one *diameter* of the *Ring* (as in *Fig. 9.*) by this it was evident that the *Satellite* was in *Apogæo*.

I observed it in *Apogæo* again on the 24th of *January* 1683, at 8^h. 00' PM. the *perpendicular* on the line of the *Ansæ* fell exactly on the *Western* limb of the *Globe* of *Saturn*, and at 9^h 30' PM. the said *perpendicular* fell within the *Globe* more than half way to the *Center*, and the distance from the line of the *Ansæ* towards the *South* seemed much about one *diameter* of the *Ring*. *Fig.* 10.

Lastly, *February* 9°. 1683. 8^h 10' PM. it was again in *Apogæo*, and I could by no means discern towards which side it enclin'd most, nor whether the transverss *diameter* of the *Ring*, or the distance of the *Satellite* therefrom were the greater; so that at that time it was precisly *Apogæon*. *Fig.* 11.

To compare with these, I chose two out of those of *Monsieur Hugens*, which seemed the most to be confided in; the first made 1659, *March* 14° *st. n.* 12^h 00'. at the *Hague*; when the *Satellite* appeared about one *diameter* of the *Ring* under *Saturn*, but it was gone so farr to the *Westward*, that he concluded, that about four hours before, or 7^h. 40'. at *London*, it had been in *Perigæo*. *Fig.* 12.

Again *March* 22°. 1659. 10^h 45'. the *Satellite* was a whole *diameter* above the line of the *Ansæ*, and the *perpendicular* thereon fell nearly upon the extremity of the *Eastern Ansæ*. See *Fig.* 6.

I could wish that we had some intermediate *Observations*, but there are none extant that I can hear of; so I proceed to the search of this *Satellit*'s period.

By the first of my *Observations* it appears that the *Satellite* was in *Perigæo* 1682 *November* 13°. 17^h 00'. *circiter*, at which time *Saturn* was 3^s. 21°. 39' from the first Star of *Aries*, in the *Ecliptick*, but the *Earth* reduced to *Saturns Equinoctial*, and the *Satellite* was 9^s. 23° 46'. a 1ª * ♈. And *March* 4° 1659. 7^h. 40'. *Saturns* place in the *Ecliptick* was 6^s. 0°. 41', but the *Earth* reduced and

consequently

consequently the *Sattllite* in 11^s. 28°. 18′ *à prima Stella Arietis*. The interval of time is 8655 daies, 9 hours, 20 minutes; in which the *Satellite* had made a certain number of *Revolutions* to the fixt Stars, and besides 9^s. 25°. 28′, or 295 *degrees*, 28 *min.* whose *Complement* to a Circle 64°. 32′, is 2 daies, 20^h, 36′ motion of the *Satellite*, according to *Hugens*. So that 8658 daies, 5^h, 56′ or 12467876 minutes of time, is the time of some number of intire *Revolutions*; and dividing that interval by 15 daies 22 hours, 39 minutes, or 22959 min. (the *Period* of *Hugens*) the *Quotient* 543 shewes the number of *Revolutions*; and again dividing 12467876 min. by 543, the 22961 $\frac{1}{2}$ min. or 15 daies, 22^h. 41′. 6″ appears to be the true time of this *Satellit's Period*. Hence the *diurnal motion* will be 22°. 34′. 38″. 18‴, and the *Annual* besides 22 *Revolutions* 10^s. 20°. 43′. Having made *Tables* to this Period, I found that in the *Apògæon* Observation of *Hugens* the *Satellite* was above 3 degrees faster than by my *calculus*, and that in the three other Observations of my own being likewise in the superior part it was about 2$\frac{1}{2}$ *degrees* slower than by the same *Calculation*. Now 'tis evident that these differenees must arise from some *Eccentricity* in the *Orbite* of this *Satellite*, and that in *March* 1659, the *Apocronion* (as I may call it,) was somewhere in the *Oriental Semicircle*, and that in *November* 1682 it was in the *Western Semicircle*, and supposing the *Apocronion* fixt, it must necessarily be between 9^s.23° 46′.and 11^s.28°. 18′. a 1^a * ♈ that being the common part between those two *Semicircles*: and because the difference was greater in *Hugen's* Observation than in *Mine*, 'twill follow that the *Linea Apsidum*, or *Apocronion*, should be nearer to 9^s. 23°. 46′. than to 11^s. 28°. 18′. I will suppose 10^s. 22°. 00′ *a prima Stella Arietis*, (which happens to be also the place of *Saturn Equinox*,) and the greatest *æquation* about 2$\frac{1}{2}$ degrees. Upon the score of this *inequality* the *mean* motion of the *Satellite* will be found about 2°. 45′ slower in 23$\frac{1}{2}$ *years*, or 7 min. in a *year*, whence I state the *Annual* motion 10^s. 20°. 36′ above 22 *Revolutions*, and the correct *Epocha* for the last day of *December* 1682 at Noon in the *Meridian* of *London* 9^s. 10°. 15′. *a* 1^a * ♈, from which Elements I compose the following *Table*.

Tabula Motus Medii Satellitis Saturnii,

ab Hugenio *inventi, a prima* ✱ ♈.

Ann. Christ. Curr.	Epochæ s. o. ′.
1641	8. 29. 17.
1661	10. 14. 10.
1681	11. 29. 3.
1682	10. 19. 39.
1683	9. 10. 15.
1684	8. 00. 51.
1685	7. 14. 2.
Mens. Ann. Com.	*Mot. Med.* s. o. ′.
Jan.	0. 0. 0.
Feb.	11. 9. 53
Mart.	8. 12. 2.
April.	7. 21. 56
Maii	6. 9. 14.
Junii	5. 19. 7.
Julii	4. 6. 26.
Aug.	3. 16. 19
Sept.	2. 26. 12.
Octob.	1. 13. 31.
Novem.	0. 23. 24.
Decem.	11. 10. 43.

Annis	*Mot. Med.* s. o. ′.
1	10. 20. 36
2	9. 11. 12
3	8. 1. 48
4	7. 14. 59
5	6. 5. 35
6	4. 26. 11
7	3. 16. 47
8	2. 26. 57
9	1. 20. 23
10	0. 11. 9
11	11. 1. 45
12	10. 14. 56
13	9. 5. 32
14	7. 26. 8
15	6. 16. 44
16	5. 29. 54
17	4. 20. 30
18	3. 11. 6
19	2. 1. 42
20	1. 14. 53

Diebus	*Mot. Med.* s. o. ′.
1	0. 32. 35
2	1. 15. 9
3	2. 7. 44
4	3. 0. 18
5	3. 22. 53
6	4. 15. 28
7	5. 8. 2
8	6. 0. 37
9	6. 23. 12
10	7. 15. 46
11	8. 8. 21
12	9. 0. 55
13	9. 23. 30
14	10. 16. 5
15	11. 8. 39
16	0. 1. 14
17	0. 23. 48
18	1. 16. 23
19	2. 8. 58
20	3. 1. 32
21	3. 24. 7
22	4. 16. 42
23	5. 9. 16
24	6. 1. 51
25	6. 24. 25
26	7. 17. 00
27	8. 9. 35
28	9. 2. 9
29	9. 24. 44
30	10. 17. 18
31	11. 9. 53
32	0. 2. 28

H. M.	*Mot. Med.* o. ′. / ′. ″.	*M.*	*Mot. Med.* ′. ″.
1	0. 56	31	29. 10
2	1. 53	32	30. 6
3	2. 49	33	31. 3
4	3. 46	34	31. 59
5	4. 42	35	32. 55
6	5. 39	36	33. 52
7	6. 35	37	34. 48
8	7. 32	38	35. 45
9	8. 28	39	36. 41
10	9. 24	40	37. 38
11	10. 21	41	38. 34
12	11. 17	42	39. 31
13	12. 14	43	40. 27
14	13. 10	44	41. 24
15	14. 7	45	42. 20
16	15. 3	46	43. 17
17	16. 0	47	44. 13
18	16. 56	48	45. 10
19	17. 52	49	46. 6
20	18. 49	50	47. 3
21	19. 45	51	47. 59
22	20. 42	52	48. 56
23	21. 38	53	49. 52
24	22. 35	54	50. 49
25	23. 31	55	51. 45
26	24. 27	56	52. 42
27	25. 24	57	53. 38
28	26. 20	58	54. 35
29	27. 17	59	55. 31
30	28. 13	60	56. 27

In Anno Bissextili post Februarium adde unum diem, motumq; ei competentem.

I here suppose the *Linea Apsidum* fixt, as having no arguments from Observation to prove the contrary, tho it be very probable that as the *Apogæon* of our *Moon* has a motion about the *Earth* in about 9 years, so that of this *Satellite* ought to have about *Saturn*, but with a much longer *Period*, which future Observation may discover.

The distance of this *Satellite* from the *Center* of *Saturn* seems to be much about 4 *Diamiters* of the *Ring*, or 9 of the *Globe*, and the plane wherein it moves very little or no thing differing from that of the *Ring*, that is to say, intersecting the *Orb* of *Saturn* 4^s. 22°. and 10^s. 22°. 41'. * ♈, with an *Angle* of $23\frac{1}{2}$ *degrees*, so as to be nearly Parallel to the *Earths Equator*; whence the *Latitude* of the *Apogæon* Semicicle from 4^s. 22°. to 10^s. 22° of *Saturns Longitude*, will be *Northern*, and of the other Semicircles *Southern*; and the contrary in the other half of *Saturns Longitude*, to wit, from 10^s. 22°. to 4^s. 22° of his distance from the first Star of ♈.

It follows now to shew how by the help of this *Table* to compute the place of this *Satellite*, to any time required.

First we must have the true *Longitude* of *Saturn* from the *Earth*, and numbred from the first Star of ♈, (or rather the place of the *Earth* viewed from *Saturn* together with its *Latitude* from the *Orb* of *Saturn*, but that being never fully $\frac{1}{7}$ of a *degree* we neglect it as a *nicety*) and therefrom *substract* 10^s. 22°. there remains the distance of *Saturn* from this *Æquinoctial* point, with which distance as with the *Longitude* of the *Sun*, take out the *Right Ascension* and *Declination* thereto ($23\frac{1}{2}$ degrees being the *obliquity* common to both) and to the *Right Ascension* adding 10^s. 22°. the summ shall be the *Longitude* of the *Satellit's Apogæon*. Then say, As *Radius* to *sine* of the *Declination*, so 8 to the greatest *Latitude* in *Apogæo*, or *Perigæo* in the parts of the *semidiamiter* of the *Ring*.

Next collect the *middle motion* of the *Satellite*, and from it *substract* 10^s. 22°; the *remainder* shall be the *mean Anomaly*, with which in the *Table* of the *Moons primary Equation*; take out the *Equation* answering thereto, and the half thereof *added* or *substracted* to or from the *middle motion*, according to the *Title*, gives the *true motion* of the *Satellite*, from which *substract* the *Apogæon*, and if the *remainder* be more than 6 *Signs*, the *Satellite* is *Occidental*; if less *Oriental*, and as *Radius* to *Sine* of the *remainder*, so 8 to the *Semidiamiters* of the *Ring*; or 18, to the *Semidiameters* of the *Globe*; that the *Satellite* is to the *Eastward* or *Westward* of the *center* of *Saturn*, according to the afore-going *Precept*.

Lastly, As *Radius*, to *Co-sine* of the said *remainder*, so is the greatest *Latitude* from the *line* of the *Ansæ*, to the *Latitude* sought.

arise-

Here Note, that I purposely neglect the *inequality* of the distance, arising from the *Eccentricity*, as being too small to be any way observable.

Lastly to clear all difficulties that may arise to them that are but little *versed* in this sort of *Calculation*, I have added Two *Examples* of the work, that where the *Precept* may seem obscure it may be thereby illustrated.

Anno 1657 *Maii* 19 *st. n. Hugens* Observed the *Satellite* very near to *Saturn* on the *Western* side, and very little above the *line* of the *Ansæ.* I suppose this about 10^h. *p. m. Anno* 1658 *Martii* 11°. 10^h *st. n.* he Observed it again, and saith of it, *difficilè conspiciebatur, quippe propinquius admodum* Saturno, *Orientem spectabat, eratq; Ansarum lineâ aliquanto inferior & quasi sub* Saturno *transiturus.* Let us now Calculate to these two times.

1657 *Maii* 9°.	9^h.	40'.	*Londini*
Saturni Locus	♍.	28 .	57'
♄ *a* 1^a ✱ ♈	5^s.	0.	32
Æquinoct. sub.	10.	22.	00
♄ *ab Æquinoct.*	6.	8.	32
Ascen. Recta.---	6.	7.	50
Apogæon.	4.	29.	50
Declin. Aust.		3.	23

1658. *Martii*	1°		10^h
Saturni Locus	♎	16°.	25'
♄ *a* 1^a ✱ ♈	5^s	17°.	59
Æquinoctium	10	22.	00
♄ *ab Æquin.*	6.	25.	59
Ascen. Recta.	6.	24.	5
Apogæon	5.	16.	5.
Declin. Aust.		10.	4

	Med.	*Mot.*	*Satel.*
1641.	8^s.	29°.	17'
16.	5.	29.	54
Maii	6.	9.	14
9.	6.	23.	12.
9^h. 40'		9.	5
Long. Med. Satel.	4.	10.	42
Apocron.	10.	22.	00
Anomalia	5.	18.	42
Æquatio sub.			31
Long. Ver. Satel.	4.	10.	11
Apogæon	4.	29.	50
Residuum	11.	10.	21
h. e. ante Apogæum		19.	39

ergo $2\frac{7}{10}$ *Semid. Annuli ad occasum & $\frac{9}{20}$ ad Boream.*

	Med.	*Mot.*	*Satel.*
1641.	8^s.	29°.	17'
17.	4.	20.	$30\frac{1}{2}$
Martii.	8.	12.	$2\frac{1}{2}$
1^d.	0	22.	$34\frac{1}{2}$
10^h		9.	$24\frac{1}{2}$
Long. Med. Satel.	11.	3.	49
Apocron.	10.	22.	00
Anomalia	0.	11.	49
Æquatio sub.		0.	30
Long. Ver. Satel.	11.	3.	19
Apogæon	5.	1.	5
Residuum	5.	17.	14
h. e. ante Perigæon		12.	46

hinc provenit $1\frac{22}{100}$ *Semid. Ann. ad ortum, & $1\frac{36}{100}$ ad Austrum sive infra.*

In each agreeing exactly with the Description and Figure of Monsieur *Hugens.*

I here call the *Plane* of this *Satellit's Orb*, which hitherto I suppose the same with that of the *Ring*, *Saturns Equinoctial*, not that any discovery hath been able to prove that the *Axis* of that *Globe* is at *right Angles* thereto, but because it has pleased Mr. *Hugens* to call it so, and likewise because it is so nearly *Parallel* to our *Globes Equinoctial*; Neverthelesss to speak my Opinion, I believe that the *Axis* is inclined, and that not a little, to the *Plane* of the *Ring*; for as the reflection of the *Suns* light from the *Ring* is a great convenience to that *Hemisphere* of *Saturn*, which beholds its illuminate side; so the other *Hemisphere* is very much incommoded by the shadow of the *Ring*, which for many *Months*, and in some *Parallels* for several *Years*, occasions a continual Night by the interception of the *Suns* beams, which is a consequence that demonstratively follows the position of the *Ring* in the *plane* of *Saturns Equator*. Now this great inconvenience would be in some measure relieved by the oblique position of the *Axis*, for then the *Parallels* of *Latitude* intersecting the *plane* of the *Ring*, many and in most cases all of them, might for some time in every *Diurnal revolution* of the *Globe* free themselves from this *Eclipse*, which otherwise were sufficient to render this *Globe* of *Saturn* unfit for any settled habitation; but this is but conjecture.

The other Two *Satellites* of *Saturn* discovered by Signor *Cassini* at *Paris Anno* 1672 and 1673, I must confess I could never yet see. I have been told that they disappear for about $\frac{2}{3}$ of *Saturns revolution*, and were only to be seen when the *Ansæ* were very *small*, it being supposed that the light which proceeds from the *Ansæ* when considerably opened might hide these *Satellits*. In the year 1685 when the *Ansæ* will be quite vanished, will be a proper time to look after them, that so we may bring their Motion to Rule, and know where to find them, for want of which knowledge 'tis likely they are at present not to be found.

Obser.